Mass moment of inertia problems and solutions pdf

Wikipedia list article Moment of inertia, denoted by I, measures the extent to which an object resists rotational acceleration about a particular axis, and is the rotational analogue to mass (which determines an object's resistance to linear acceleration). Mass moments of inertia have units of dimension ML2([mass] × [length]2). It should not be confused with the second moment of area, which is used in beam calculations. The mass moment of inertia is often also known as the rotational inertia, and sometimes as the angular mass. For simple objects with geometric symmetry, one can often determine the moment of inertia in an exact closed-form expression. Typically this occurs when the mass density is constant, but in some cases the density can vary throughout the object as well. In general, it may not be straightforward to symbolically express the moment of inertia of shapes with more complicated mass distributions and lacking symmetry. When calculating moments of inertia, it is useful to remember that it is an additive function and exploit the parallel axis and perpendicular axis theorems. This article mainly considers symmetric mass distributions, with constant density throughout the object, and the axis of rotation is taken to be through the center of mass unless otherwise specified. Moments of inertia Following are scalar moments of inertia. In general, the moment of inertia is a tensor, see below. Description Figure Moment(s) of inertia Point mass M at a distance r from the axis of rotation. A point mass does not have a moment of inertia around its own axis, but using the parallel axis theorem a moment of inertia around a distant axis of rotation is achieved. I = M r 2 {\displaystyle I=Mr^{2}} Two point masses, m1 and m2, with reduced mass μ and separated by a distance x, about an axis passing through the center of mass of the system and perpendicular to the line joining the two particles. I = m 1 m 2 m 1 + m 2 x 2 = μ x 2 {\displaystyle I={\frac {m_{1}m_{2}}{m_{1}\!+\!m_{2}}}x^{2}=\mu x^{2}} Thin rod of length L and mass m, perpendicular to the axis of rotation, rotating about its center. This expression assumes that the rod is an infinitely thin (but rigid) wire. This is a special case of the thin rectangular plate with axis of rotation at the center of the plate, with w = L and h = 0. I c e n t e r = 1 12 m L 2 {\displaystyle I_{\mathrm {center} }={\frac {1}{12}}mL^{2}\,\!} [1] Thin rod of length L and mass m, perpendicular to the axis of rotation, rotating about one end. This expression assumes that the rod is an infinitely thin (but rigid) wire. This is also a special case of the thin rectangular plate with axis of rotation at the end of the plate, with h = L and w = 0. I e n d = 1 3 m L 2 {\displaystyle I_{\mathrm {end} }={\frac {1}{3}}mL^{2}\,\!} [1] Thin circular loop of radius r and mass m. This is a special case of a torus for a = 0 (see below), as well as of a thick-walled cylindrical tube with open ends, with r1 = r2 and h = 0. I z = m r 2 {\displaystyle I_{z}=mr^{2}\!} I x = I y = 1 2 m r 2 {\displaystyle I_{x}=I_{y}={\frac {1}{2}}mr^{2}\,\!} Thin, solid disk of radius r and mass m. This is a special case of the solid cylinder, with h = 0. That I x = I y = I z 2 {\displaystyle I_{x}=I_{y}={\frac {I_{z}}{2}}\,} is a consequence of the perpendicular axis theorem. I z = 1 2 m r 2 {\displaystyle I_{z}={\frac {1}{2}}mr^{2}\,\!} I x = I y = 1 4 m r 2 {\displaystyle I_{x}=I_{y}={\frac {1}{4}}mr^{2}\,\!} A uniform annulus (disk with a concentric hole) of mass m, inner radius r1 and outer radius r2 I = 1 2 m (r 1 2 + r 2 2) {\displaystyle I={\frac {1}{2}}m(r_{1}^{2}+r_{2}^{2})} An annulus with a constant area density ρ A {\displaystyle \rho _{A}} I = 1 2 π ρ A (r 2 4 − r 1 4) {\displaystyle I={\frac {1}{2}}\pi \rho _{A}(r_{2}^{4}-r_{1}^{4})} Thin cylindrical shell with open ends, of radius r and mass m. This expression assumes that the shell thickness is negligible. It is a special case of the thick-walled cylindrical tube for r1 = r2. Also, a point mass m at the end of a rod of length r has this same moment of inertia and the value r is called the radius of gyration. I ≈ m r 2 {\displaystyle I\approx mr^{2}\,\!} [1] Solid cylinder of radius r, height h and mass m. This is a special case of the thick-walled cylindrical tube, with r1 = 0. I z = 1 2 m r 2 {\displaystyle I_{z}={\frac {1}{2}}mr^{2}\,\!} [1] I x = I y = 1 12 m (3 r 2 + h 2) {\displaystyle I_{x}=I_{y}={\frac {1}{12}}m\left(3r^{2}+h^{2}\right)} Thick-walled cylindrical tube with open ends, of inner radius r1, outer radius r2, length h and mass m. I z = 1 2 m (r 2 2 + r 1 2) = m r 2 2 (1 − t + t 2 2) {\displaystyle I_{z}={\frac {1}{2}}m\left(r_{2}^{2}+r_{1}^{2}\right)=mr_{2}^{2}\left(1-t+{\frac {t^{2}}{2}}\right)} [1] [2] where t = (r2 − r1)/r2 is a normalized thickness ratio; I x = I y = 1 12 m (3 (r 2 2 + r 1 2) + h 2) {\displaystyle I_{x}=I_{y}={\frac {1}{12}}m\left(3\left(r_{2}^{2}+r_{1}^{2}\right)+h^{2}\right)} The above formula is for the xy plane being at the middle of the cylinder. If the xy plane is at the base of the cylinder, then the following formula applies: I x = I y = 1 12 m (3 (r 2 2 + r 1 2) + 4 h 2) {\displaystyle I_{x}=I_{y}={\frac {1}{12}}m\left(3\left(r_{2}^{2}+r_{1}^{2}\right)+4h^{2}\right)} With a density of ρ and the same geometry note: this is for an object with a constant density I z = π ρ h 2 (r 2 4 − r 1 4) {\displaystyle I_{z}={\frac {\pi \rho h}{2}}\left(r_{2}^{4}-r_{1}^{4}\right)} I x = I y = π ρ h 12 (3 (r 2 4 − r 1 4) + h 2 (r 2 2 − r 1 2)) {\displaystyle I_{x}=I_{y}={\frac {\pi \rho h}{12}}\left(3(r_{2}^{4}-r_{1}^{4})+h^{2}(r_{2}^{2}-r_{1}^{2})\right)} Regular tetrahedron of side s and mass m I s o l i d = 1 20 m s 2 {\displaystyle I_{\mathrm {solid} }={\frac {1}{20}}ms^{2}\,\!} I h o l l o w = 1 12 m s 2 {\displaystyle I_{\mathrm {hollow} }={\frac {1}{12}}ms^{2}\,\!} [3] Regular octahedron of side s and mass m I x , h o l l o w = I y , h o l l o w = I z , h o l l o w = 1 6 m s 2 {\displaystyle I_{x,\mathrm {hollow} }=I_{y,\mathrm {hollow} }=I_{z,\mathrm {hollow} }={\frac {1}{6}}ms^{2}\,\!} [3] I x , s o l i d = I y , s o l i d = I z , s o l i d = 1 10 m s 2 {\displaystyle I_{x,\mathrm {solid} }=I_{y,\mathrm {solid} }=I_{z,\mathrm {solid} }={\frac {1}{10}}ms^{2}\,\!} [3] Regular dodecahedron of side s and mass m I x , h o l l o w = I y , h o l l o w = I z , h o l l o w = 39 ϕ + 28 90 m s 2 {\displaystyle I_{x,\mathrm {hollow} }=I_{y,\mathrm {hollow} }=I_{z,\mathrm {hollow} }={\frac {39\phi +28}{90}}ms^{2}} I x , s o l i d = I y , s o l i d = I z , s o l i d = 39 ϕ + 28 150 m s 2 {\displaystyle I_{x,\mathrm {solid} }=I_{y,\mathrm {solid} }=I_{z,\mathrm {solid} }={\frac {39\phi +28}{150}}ms^{2}\,\!} (where ϕ = 1 + 5 2 {\displaystyle \phi ={\frac {1+{\sqrt {5}}}{2}}}) [3] Regular icosahedron of side s and mass m I x , h o l l o w = I y , h o l l o w = I z , h o l l o w = ϕ 2 6 m s 2 {\displaystyle I_{x,\mathrm {hollow} }=I_{y,\mathrm {hollow} }=I_{z,\mathrm {hollow} }={\frac {\phi ^{2}}{6}}ms^{2}} I x , s o l i d = I y , s o l i d = I z , s o l i d = ϕ 2 10 m s 2 {\displaystyle I_{x,\mathrm {solid} }=I_{y,\mathrm {solid} }=I_{z,\mathrm {solid} }={\frac {\phi ^{2}}{10}}ms^{2}\,\!} [3] Hollow sphere of radius r and mass m. I = 2 3 m r 2 {\displaystyle I={\frac {2}{3}}mr^{2}\,\!} [1] Solid sphere (ball) of radius r and mass m. I = 2 5 m r 2 {\displaystyle I={\frac {2}{5}}mr^{2}\,\!} [1] Sphere (shell) of radius r2 and mass m, with centered spherical cavity of radius r1. When the cavity radius r1 = 0, the object is a solid ball (above). When r1 = r2, (r 2 5 − r 1 5 r 2 3 − r 1 3) = 5 3 r 2 2 {\displaystyle \left({\frac {r_{2}^{5}-r_{1}^{5}}{r_{2}^{3}-r_{1}^{3}}}\right)={\frac {5}{3}}r_{2}^{2}} , and the object is a hollow sphere. I = 2 5 m (r 2 5 − r 1 5 r 2 3 − r 1 3) {\displaystyle I={\frac {2}{5}}m\left({\frac {r_{2}^{5}-r_{1}^{5}}{r_{2}^{3}-r_{1}^{3}}}\right)\,\!} [1] Right circular cone with radius r, height h and mass m I z = 3 10 m r 2 {\displaystyle I_{z}={\frac {3}{10}}mr^{2}\,\!} [4] About an axis passing through the tip: I x = I y = m (3 20 r 2 + 3 5 h 2) {\displaystyle I_{x}=I_{y}=m\left({\frac {3}{20}}r^{2}+{\frac {3}{5}}h^{2}\right)\,\!} [4] About an axis passing through the base: I x = I y = m (3 20 r 2 + 1 10 h 2) {\displaystyle I_{x}=I_{y}=m\left({\frac {3}{20}}r^{2}+{\frac {1}{10}}h^{2}\right)\,\!} About an axis passing through the center of mass: I x = I y = m (3 20 r 2 + 3 80 h 2) {\displaystyle I_{x}=I_{y}=m\left({\frac {3}{20}}r^{2}+{\frac {3}{80}}h^{2}\right)\,\!} Right circular hollow cone with radius r, height h and mass m I z = 1 2 m r 2 {\displaystyle I_{z}={\frac {1}{2}}mr^{2}\,\!} [4] I x = I y = 1 4 m (r 2 + 2 h 2) {\displaystyle I_{x}=I_{y}={\frac {1}{4}}m\left(r^{2}+2h^{2}\right)\,\!} [4] Torus with minor radius a, major radius b and mass m. About an axis passing through the center and perpendicular to the diameter: 1 4 m (4 b 2 + 3 a 2) {\displaystyle {\frac {1}{4}}m\left(4b^{2}+3a^{2}\right)} [5]About a diameter: 1 8 m (5 a 2 + 4 b 2) {\displaystyle {\frac {1}{8}}m\left(5a^{2}+4b^{2}\right)} [5] Ellipsoid (solid) of semiaxes a, b, and c with mass m I a = 1 5 m (b 2 + c 2) {\displaystyle I_{a}={\frac {1}{5}}m\left(b^{2}+c^{2}\right)\,\!} I b = 1 5 m (a 2 + c 2) {\displaystyle I_{b}={\frac {1}{5}}m\left(a^{2}+c^{2}\right)\,\!} I c = 1 5 m (a 2 + b 2) {\displaystyle I_{c}={\frac {1}{5}}m\left(a^{2}+b^{2}\right)\,\!} Thin rectangular plate of height h, width w and mass m(Axis of rotation at the end of the plate) I e = 1 12 m (4 h 2 + w 2) {\displaystyle I_{e}={\frac {1}{12}}m\left(4h^{2}+w^{2}\right)\,\!} Thin rectangular plate of height h, width w and mass m(Axis of rotation at the center) I c = 1 12 m (h 2 + w 2) {\displaystyle I_{c}={\frac {1}{12}}m\left(h^{2}+w^{2}\right)\,\!} [1] Thin rectangular plate of radius r[a] and mass m (Axis of rotation along a side of the plate) I = 1 3 m r 2 {\displaystyle I={\frac {1}{3}}mr^{2}} Solid cuboid of height h, width w, and depth d, and mass m. For a similarly oriented cube with sides of length s {\displaystyle s} , I C M = 1 6 m s 2 {\displaystyle I_{\mathrm {CM} }={\frac {1}{6}}ms^{2}\,\!} I h = 1 12 m (w 2 + d 2) {\displaystyle I_{h}={\frac {1}{12}}m\left(w^{2}+d^{2}\right)} I w = 1 12 m (d 2 + h 2) {\displaystyle I_{w}={\frac {1}{12}}m\left(d^{2}+h^{2}\right)} I d = 1 12 m (w 2 + h 2) {\displaystyle I_{d}={\frac {1}{12}}m\left(w^{2}+h^{2}\right)} Solid cuboid of height D, width W, and length L, and mass m, rotating about the longest diagonal. For a cube with sides s {\displaystyle s} , I = 1 6 m s 2 {\displaystyle I={\frac {1}{6}}ms^{2}\,\!} . I = 1 6 m (W 2 D 2 + D 2 L 2 + W 2 L 2 W 2 + D 2 + L 2) {\displaystyle I={\frac {1}{6}}m\left({\frac {W^{2}D^{2}+D^{2}L^{2}+W^{2}L^{2}}{W^{2}+D^{2}+L^{2}}}\right)} Tilted solid cuboid of depth d, width w, and length l, and mass m, rotating about the vertical axis (axis y as seen in figure). For a cube with sides s {\displaystyle s} , I = 1 6 m s 2 {\displaystyle I={\frac {1}{6}}ms^{2}\,\!} . I = m 12 (l 2 cos 2 ⁡ β + d 2 sin 2 ⁡ β + w 2) {\displaystyle I={\frac {m}{12}}\left(l^{2}\cos ^{2}\beta +d^{2}\sin ^{2}\beta +w^{2}\right)} [6] Triangle with vertices at the origin and at P and Q, with mass m, rotating about an axis perpendicular to the plane and passing through the origin. I = 1 6 m (P · P + P · Q + Q · Q) {\displaystyle I={\frac {1}{6}}m(\mathbf {P} \cdot \mathbf {P} +\mathbf {P} \cdot \mathbf {Q} +\mathbf {Q} \cdot \mathbf {Q})} Plane polygon with vertices P1, P2, P3, ..., PN and mass m uniformly distributed on its interior, rotating about an axis perpendicular to the plane and passing through the origin. I = m (∑ n = 1 N ‖ P n + 1 × P n ‖ ((P n · P n) + (P n · P n + 1) + (P n + 1 · P n + 1)) 6 ∑ n = 1 N ‖ P n + 1 × P n ‖) {\displaystyle I=m\left({\frac {\sum \limits _{n=1}^{N}\|\mathbf {P} _{n+1}\times \mathbf {P} _{n}\|\left(\left(\mathbf {P} _{n}\cdot \mathbf {P} _{n}\right)+\left(\mathbf {P} _{n}\cdot \mathbf {P} _{n+1}\right)+\left(\mathbf {P} _{n+1}\cdot \mathbf {P} _{n+1}\right)\right)}{6\sum \limits _{n=1}^{N}\|\mathbf {P} _{n+1}\times \mathbf {P} _{n}\|}}\right)} Plane regular polygon with n-vertices and mass m uniformly distributed on its interior, rotating about an axis perpendicular to the plane and passing through its barycenter. R is the radius of the circumscribed circle. I = 1 2 m R 2 (1 − 2 3 sin 2 ⁡ (π n)) {\displaystyle I={\frac {1}{2}}mR^{2}\left(1-{\frac {2}{3}}\sin ^{2}\left({\tfrac {\pi }{n}}\right)\right)} [7] An isosceles triangle of mass M, vertex angle 2β and common-side length L (axis through tip, perpendicular to plane) I = 1 2 m L 2 (1 − 2 3 sin 2 ⁡ (β)) {\displaystyle I={\frac {1}{2}}mL^{2}\left(1-{\frac {2}{3}}\sin ^{2}\left(\beta \right)\right)} [7] Infinite disk with mass distributed in a Bivariate Gaussian distribution on two axes around the axis of rotation with mass-density as a function of the position vector x {\displaystyle {\mathbf {x} }} ρ (x) = m exp ⁡ (− 1 2 x T Σ − 1 x) (2 π) 2 | Σ | {\displaystyle \rho ({\mathbf {x} })=m{\frac {\exp \left(-{\frac {1}{2}}{\mathbf {x} }^{\mathrm {T} }{\boldsymbol {\Sigma }}^{-1}{\mathbf {x} }\right)}{\sqrt {(2\pi)^{2}|{\boldsymbol {\Sigma }}|}}}} I = m · tr ⁡ (Σ) {\displaystyle I=m\cdot \operatorname {tr} ({\boldsymbol {\Sigma }})\,\!} List of 3D inertia tensors This list of moment of inertia tensors is given for principal axes of each object. To obtain the scalar moments of inertia I above, the tensor moment of inertia I is projected along some axis defined by a unit vector n according to the formula: n · I · n ≡ n i I i j n j , {\displaystyle \mathbf {n} \cdot \mathbf {I} \cdot \mathbf {n} \equiv n_{i}I_{ij}n_{j}\,,} where the dots indicate tensor contraction and the Einstein summation convention is used. In the above table, n would be the unit Cartesian basis ex, ey, ez to obtain Ix, Iy, Iz respectively. Description Figure Moment of inertia tensor Solid sphere of radius r and mass m I = [2 5 m r 2 0 0 0 2 5 m r 2 0 0 0 2 5 m r 2] {\displaystyle I={\begin{bmatrix}{\frac {2}{5}}mr^{2}&0&0\\0&{\frac {2}{5}}mr^{2}&0\\0&0&{\frac {2}{5}}mr^{2}\end{bmatrix}}} Hollow sphere of radius r and mass m I = [2 3 m r 2 0 0 0 2 3 m r 2 0 0 0 2 3 m r 2] {\displaystyle I={\begin{bmatrix}{\frac {2}{3}}mr^{2}&0&0\\0&{\frac {2}{3}}mr^{2}&0\\0&0&{\frac {2}{3}}mr^{2}\end{bmatrix}}} Solid ellipsoid of semi-axes a, b, c and mass m I = [1 5 m (b 2 + c 2) 0 0 0 1 5 m (a 2 + c 2) 0 0 0 1 5 m (a 2 + b 2)] {\displaystyle I={\begin{bmatrix}{\frac {1}{5}}m(b^{2}+c^{2})&0&0\\0&{\frac {1}{5}}m(a^{2}+c^{2})&0\\0&0&{\frac {1}{5}}m(a^{2}+b^{2})\end{bmatrix}}} Right circular cone with radius r, height h and mass m, about the apex I = [3 5 m h 2 + 3 20 m r 2 0 0 0 3 5 m h 2 + 3 20 m r 2 0 0 0 3 10 m r 2] {\displaystyle I={\begin{bmatrix}{\frac {3}{5}}mh^{2}+{\frac {3}{20}}mr^{2}&0&0\\0&{\frac {3}{5}}mh^{2}+{\frac {3}{20}}mr^{2}&0\\0&0&{\frac {3}{10}}mr^{2}\end{bmatrix}}} Solid cuboid of width w, height h, depth d, and mass m I = [1 12 m (h 2 + d 2) 0 0 0 1 12 m (w 2 + d 2) 0 0 0 1 12 m (w 2 + h 2)] {\displaystyle I={\begin{bmatrix}{\frac {1}{12}}m(h^{2}+d^{2})&0&0\\0&{\frac {1}{12}}m(w^{2}+d^{2})&0\\0&0&{\frac {1}{12}}m(w^{2}+h^{2})\end{bmatrix}}} Slender rod along y-axis of length l and mass m about end I = [1 3 m l 2 0 0 0 0 0 0 0 1 3 m l 2] {\displaystyle I={\begin{bmatrix}{\frac {1}{3}}ml^{2}&0&0\\0&0&0\\0&0&{\frac {1}{3}}ml^{2}\end{bmatrix}}} Slender rod along y-axis of length l and mass m about center I = [1 12 m l 2 0 0 0 0 0 0 0 1 12 m l 2] {\displaystyle I={\begin{bmatrix}{\frac {1}{12}}ml^{2}&0&0\\0&0&0\\0&0&{\frac {1}{12}}ml^{2}\end{bmatrix}}} Solid cylinder of radius r, height h and mass m I = [1 12 m (3 r 2 + h 2) 0 0 0 1 12 m (3 r 2 + h 2) 0 0 0 1 2 m r 2] {\displaystyle I={\begin{bmatrix}{\frac {1}{12}}m(3r^{2}+h^{2})&0&0\\0&{\frac {1}{12}}m(3r^{2}+h^{2})&0\\0&0&{\frac {1}{2}}mr^{2}\end{bmatrix}}} Thick-walled cylindrical tube with open ends, of inner radius r1, outer radius r2, length h and mass m I = [1 12 m (3 (r 2 2 + r 1 2) + h 2) 0 0 0 1 12 m (3 (r 2 2 + r 1 2) + h 2) 0 0 0 1 2 m (r 2 2 + r 1 2)] {\displaystyle I={\begin{bmatrix}{\frac {1}{12}}m(3(r_{2}^{2}+r_{1}^{2})+h^{2})&0&0\\0&{\frac {1}{12}}m(3(r_{2}^{2}+r_{1}^{2})+h^{2})&0\\0&0&{\frac {1}{2}}m(r_{2}^{2}+r_{1}^{2})\end{bmatrix}}} See also List of second moments of area Parallel axis theorem Perpendicular axis theorem Notes ^ Width perpendicular to the axis of rotation (side of plate); height (parallel to axis) is irrelevant. 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Introduction to Classical Mechanics: With Problems and Solutions; first edition (8 January 2010). Cambridge University Press. p. 320. ISBN 978-0521876223. External links The inertia tensor of a tetrahedron Tutorial on deriving moment of inertia for common shapes Retrieved from "

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